Lesson 5

Thermomechanical Measurements for Energy Systems (MENR)

Measurements for Mechanical Systems and Production (MMER)

A.Y. 2015-16

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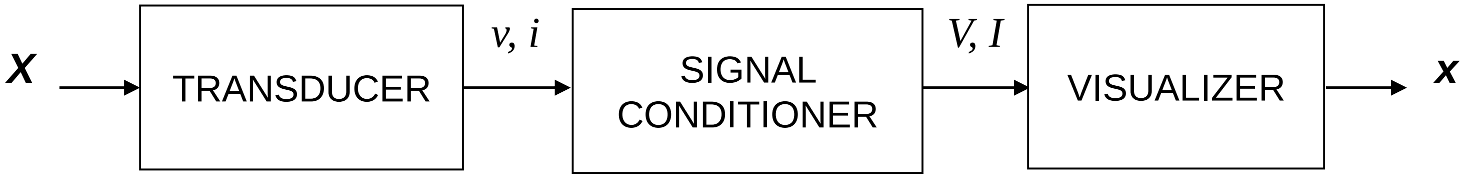
We will now analyze more in depth each one of the *functional blocks* of the general ***measurement chain*** …

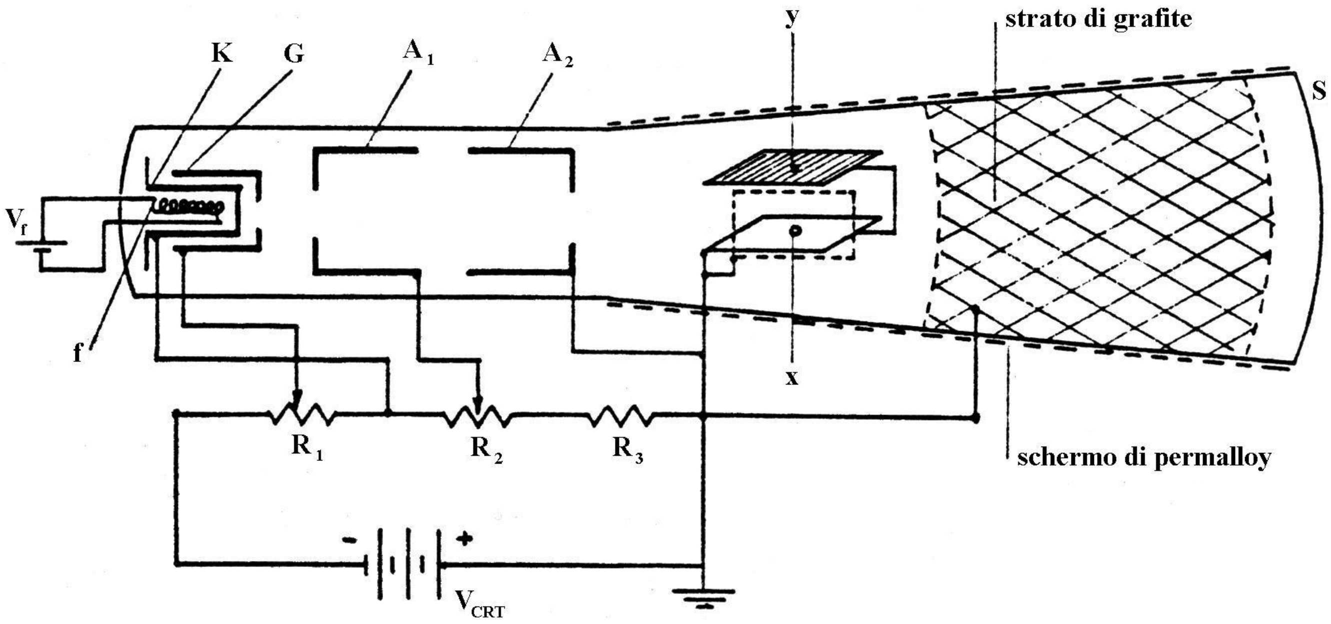


In the industry and in most modern measurement applications, the *measurement chain is electric* and the *information* passes through the different stages as ***voltage signal*** or as a ***current !***

Despite the enormous technological advancements of the last 30 years, the main *electrical visualizer instruments* can still be included in only **three** classes:

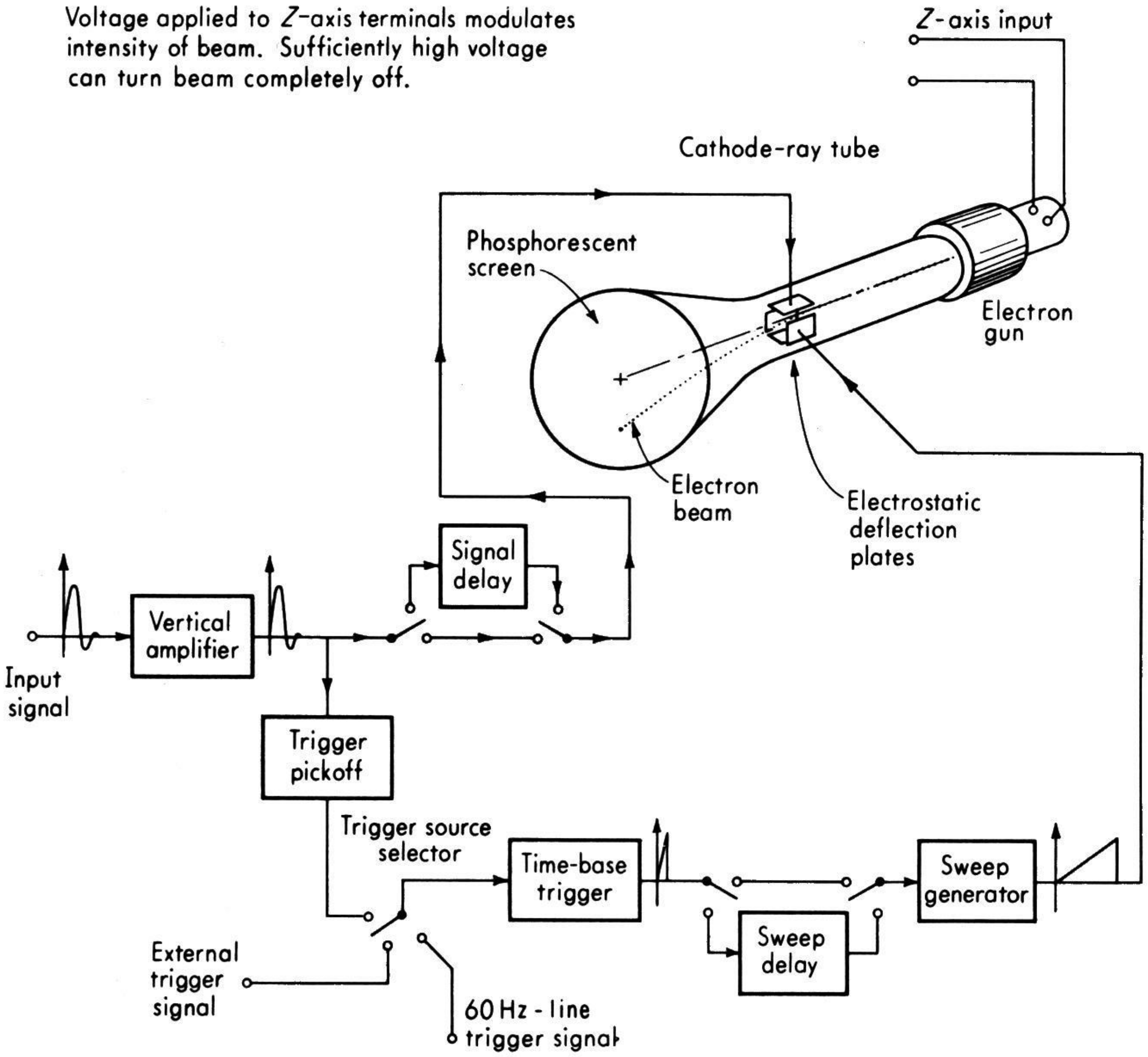
1. The ***galvanometer***
2. The ***oscilloscope***
3. The ***digital voltmeter***

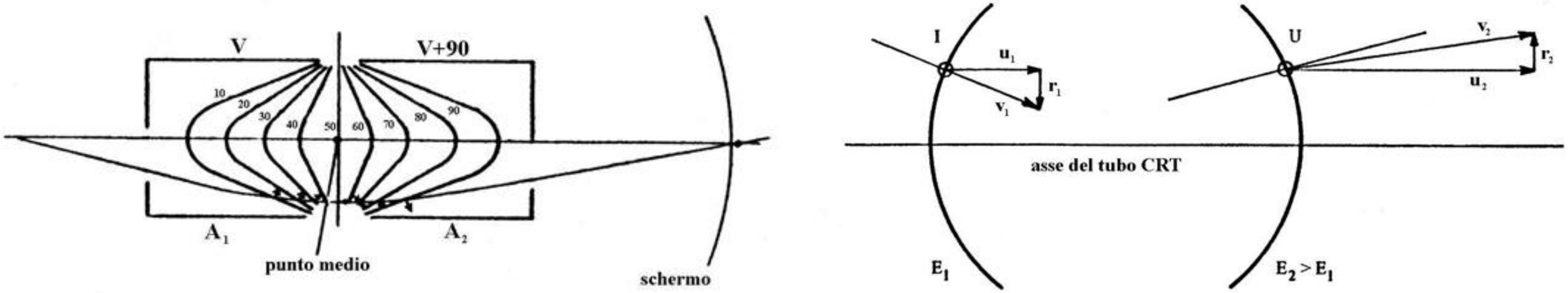
The analog OSCILLOSCOPE →



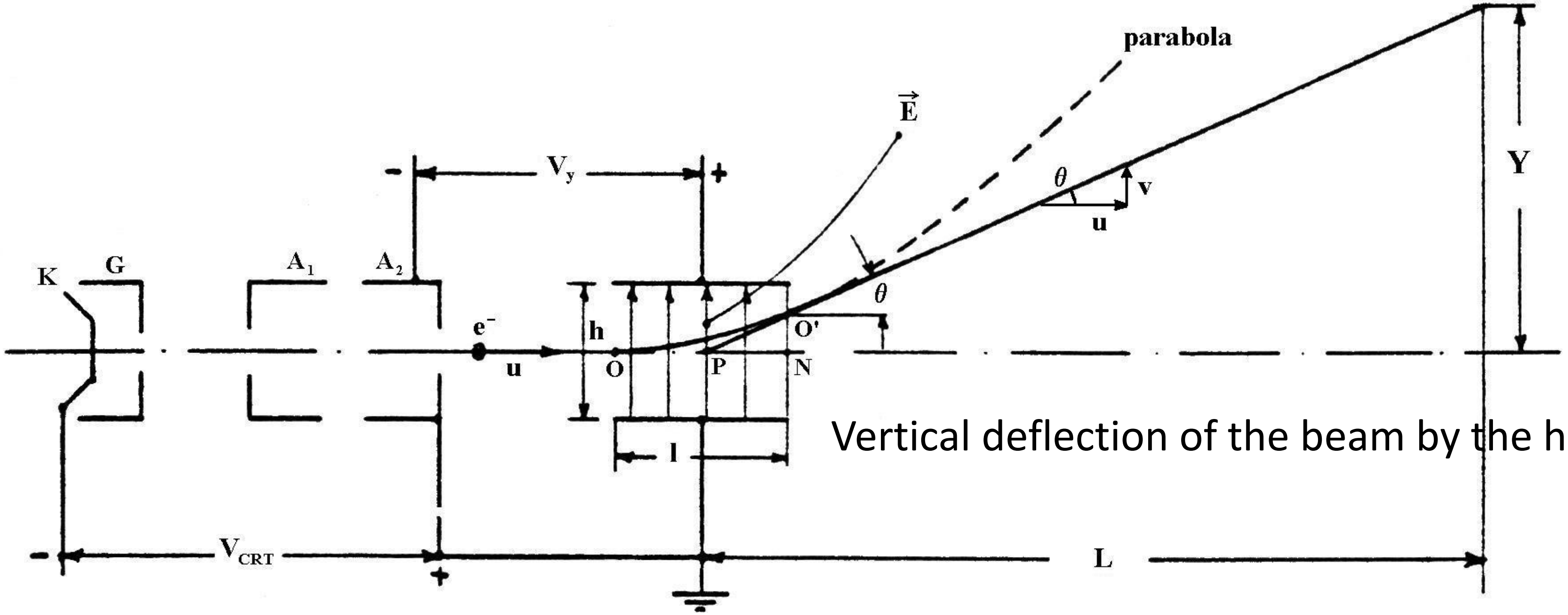
Analog oscilloscope is the older type and can <u>*instantaneously*</u> display on the screen the waveform of the input signal !
The *periodic input signal* is combined on the fluorescent screen with an *internal sawtooth (sweep) signal* !

A ***trigger*** synchronizes the two signals !

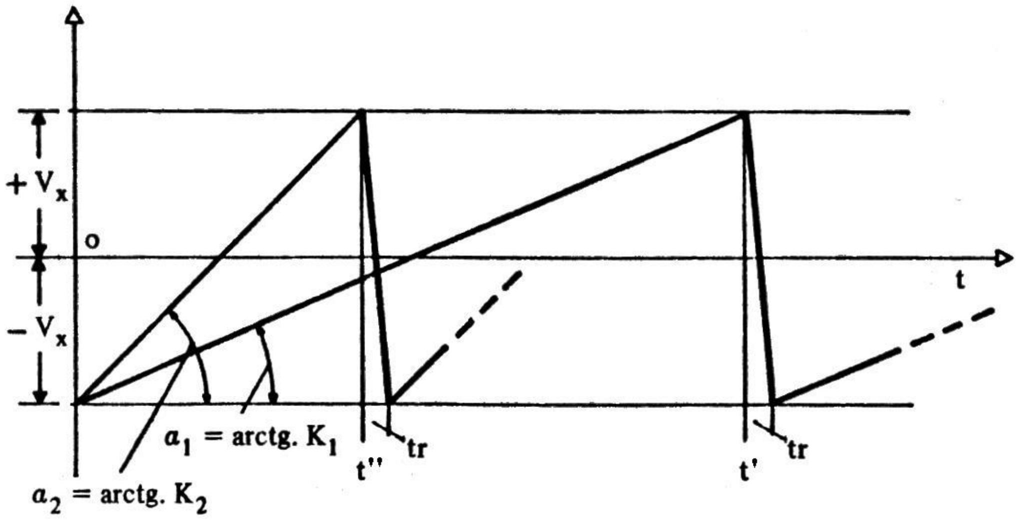




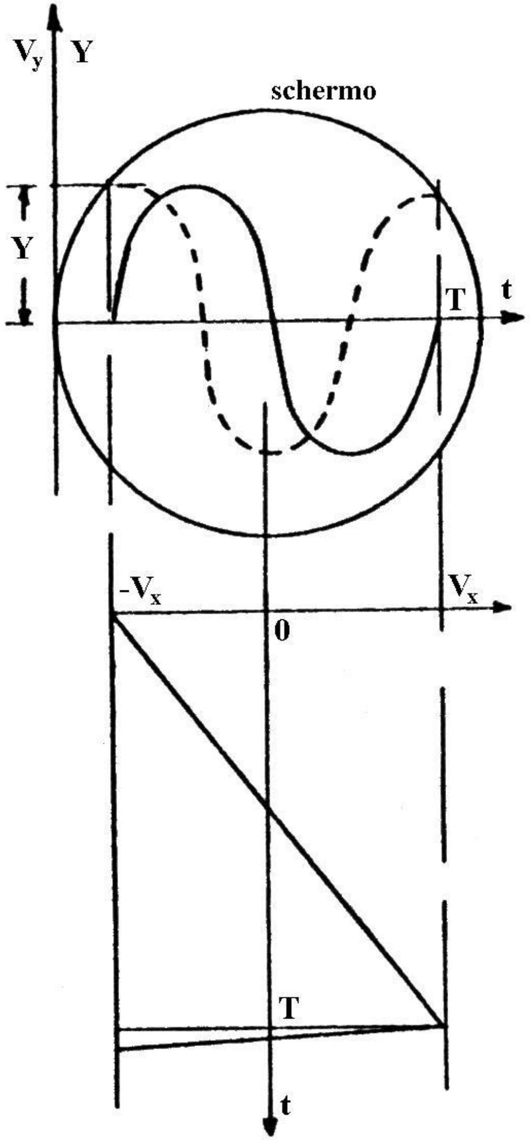
Acceleration and *collimation* of the electron beam !

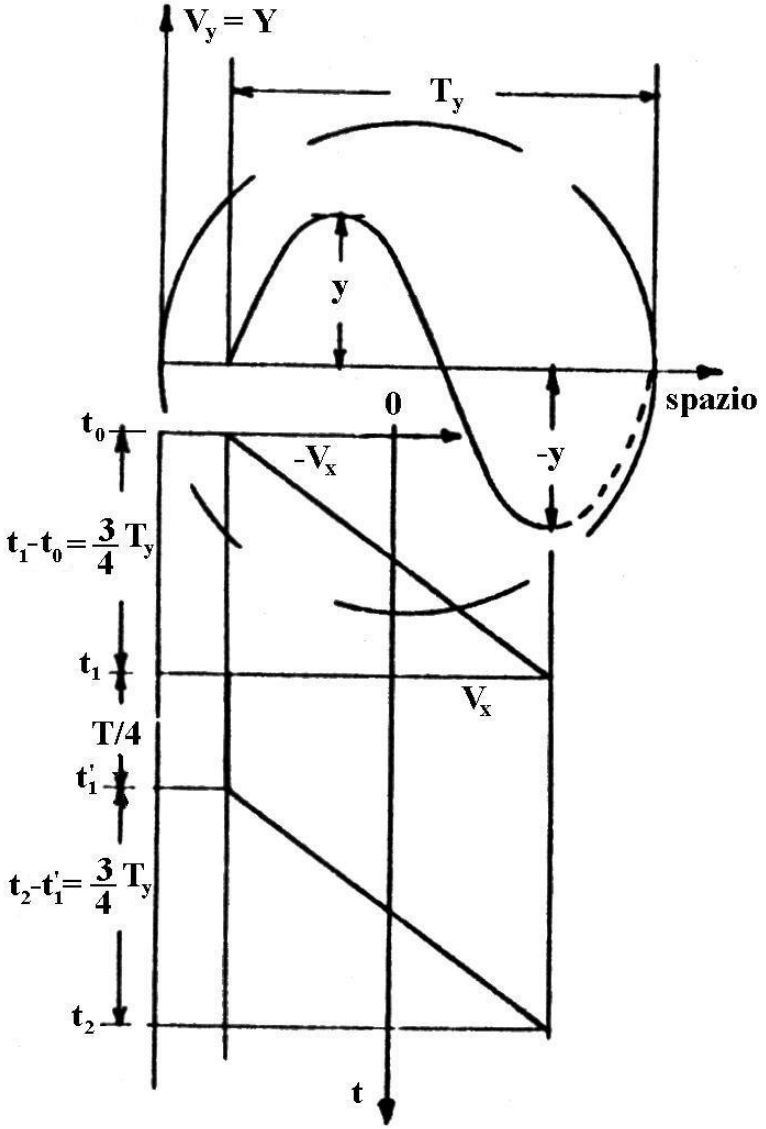


Vertical deflection of the beam by the horizontal plates !

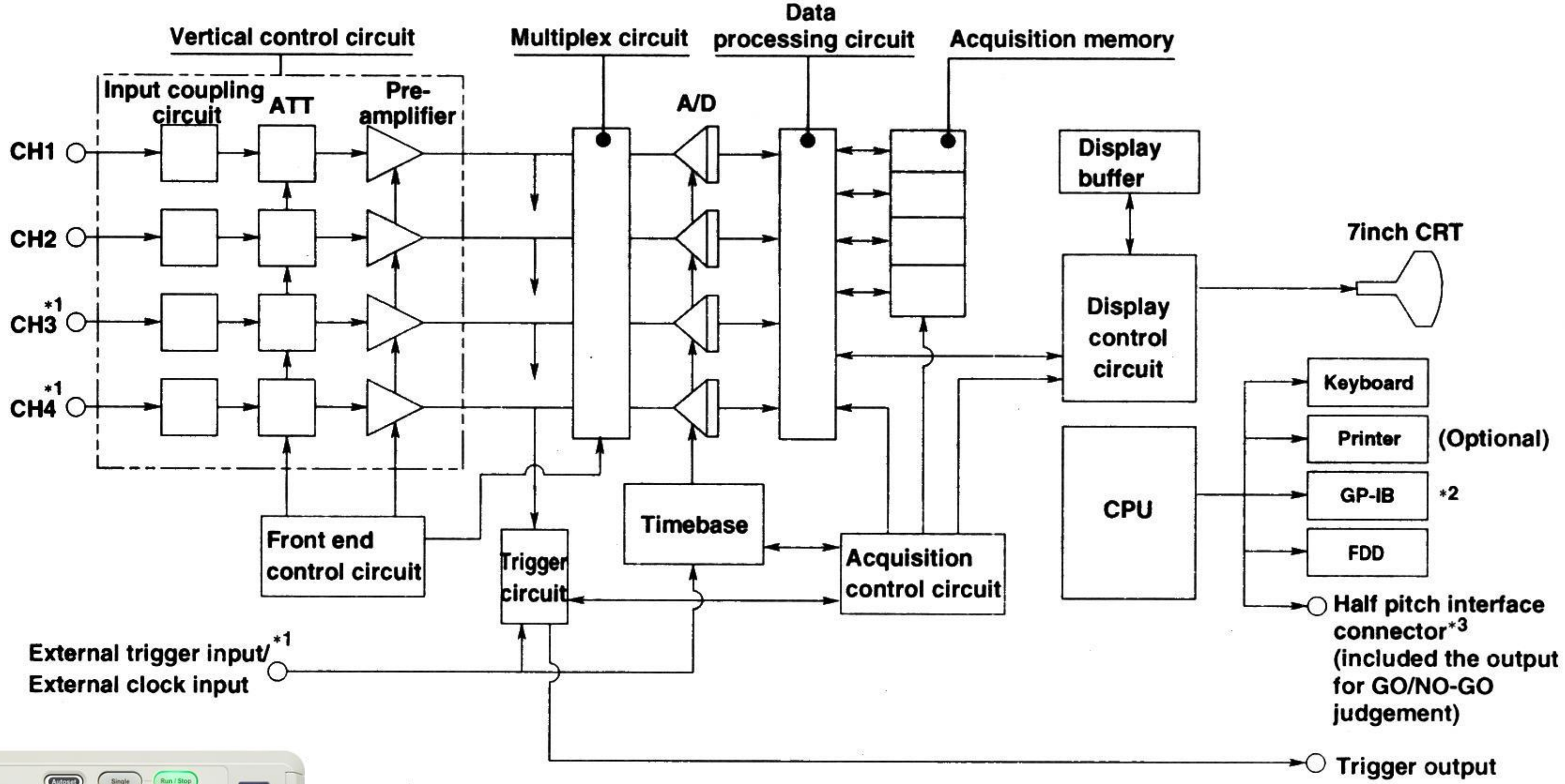


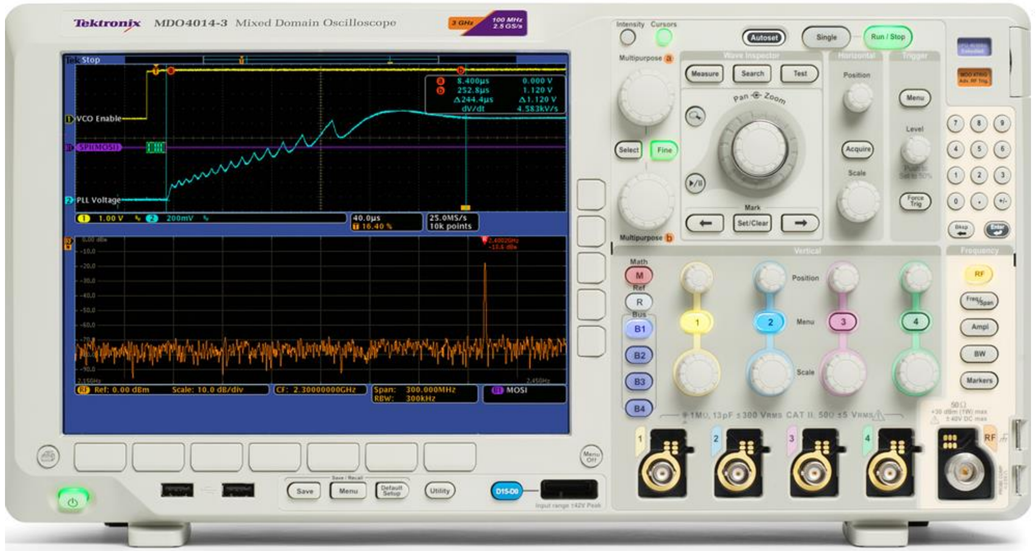
Sawtooth (sweep) and *sinusoidal signal* combination on the CRT screen !





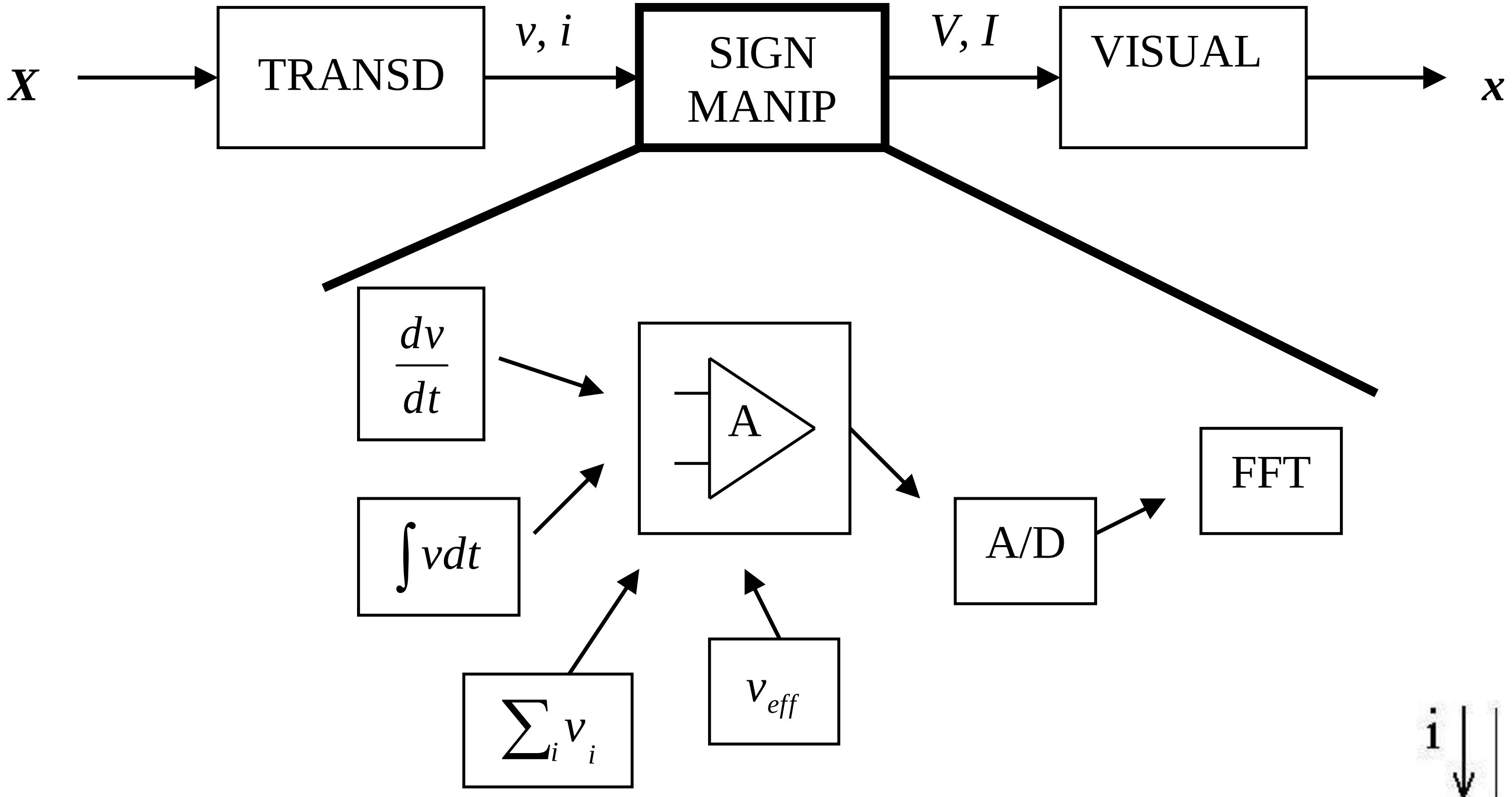
Digital OSCILLOSCOPE …





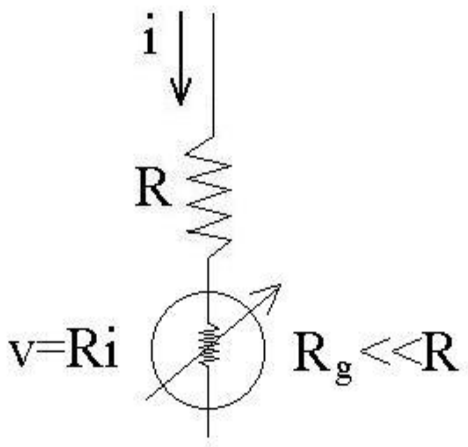
The digital oscilloscope does the same tasks as the *analog* scope (and much more) but it is actually a ***digital voltmeter*** that samples the input signal and displays it on a VGA screen !

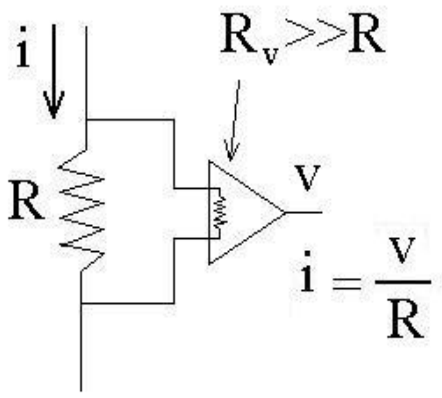
We will now study the most important ***signal manipulation circuits*** used in measurement instruments :



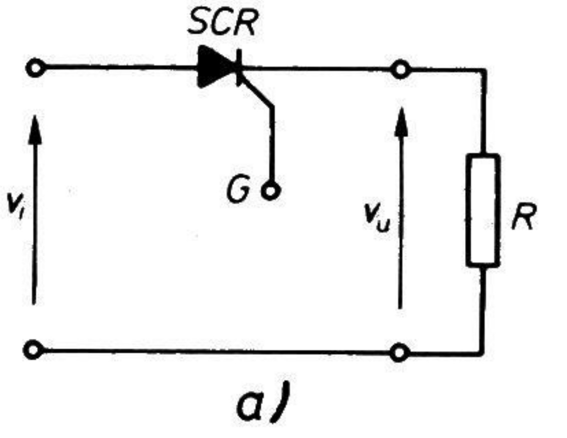
In modern and complex industry measurement system, the signal outputted by the transducer is almost always an *electrical signal* ***v*** *or* ***i*** *for* which, most of the time, we need to *enhance the information content* (the ***intensity of the measurand***) !

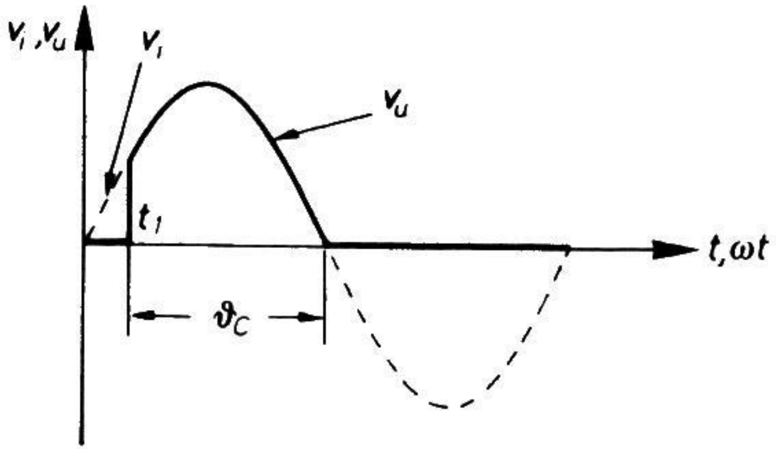
It is always possible to measure a voltage with an amperometer and a current with a voltmeter …



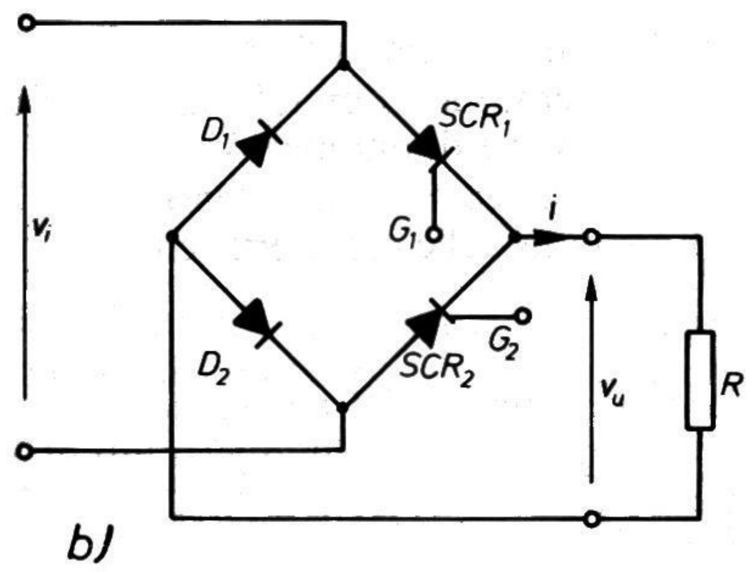


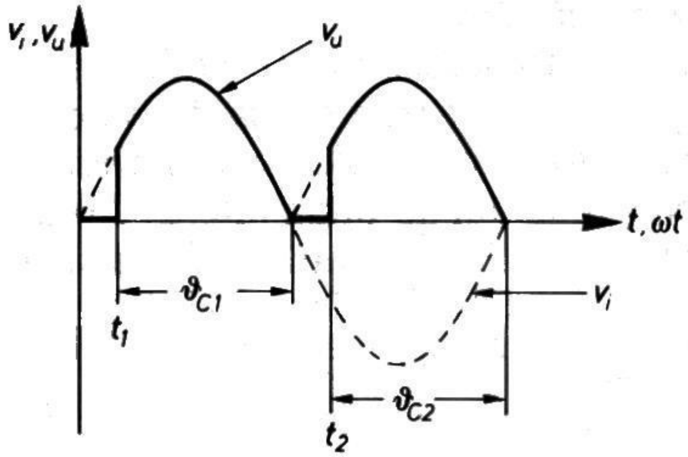
The rectifier diode :





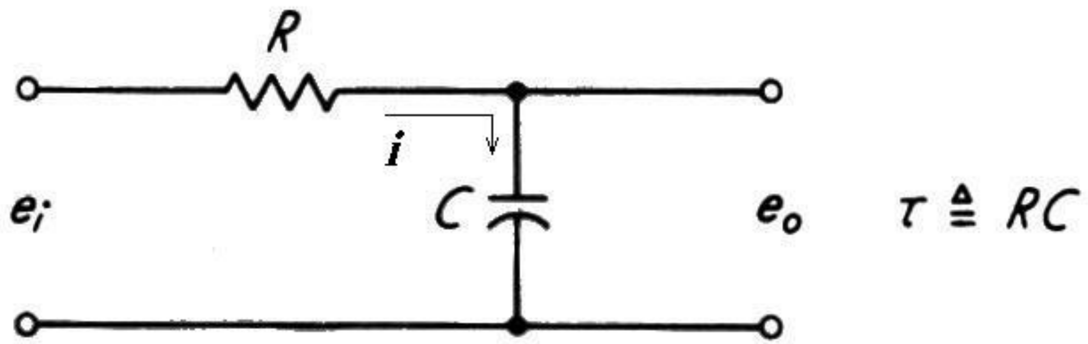
a) The ***SCR diode*** works here as a *half-wave rectifier* !





b) The ***Gretz bridge*** with 4 SCR diodes works here as a *full- wave rectifier* !

The low-pass "RC" circuit



Let's study the *two electric circuits*

input (voltage) signal: $v_i = (R + X_C) \cdot i$

output (voltage) signal: $v_o = X_C \cdot i$

the current is $i = \dfrac{v_i}{R + X_C}$ and the output voltage: $v_o = X_C \cdot \dfrac{v_i}{R + X_C} = \dfrac{v_i}{\dfrac{R}{X_C} + 1}$

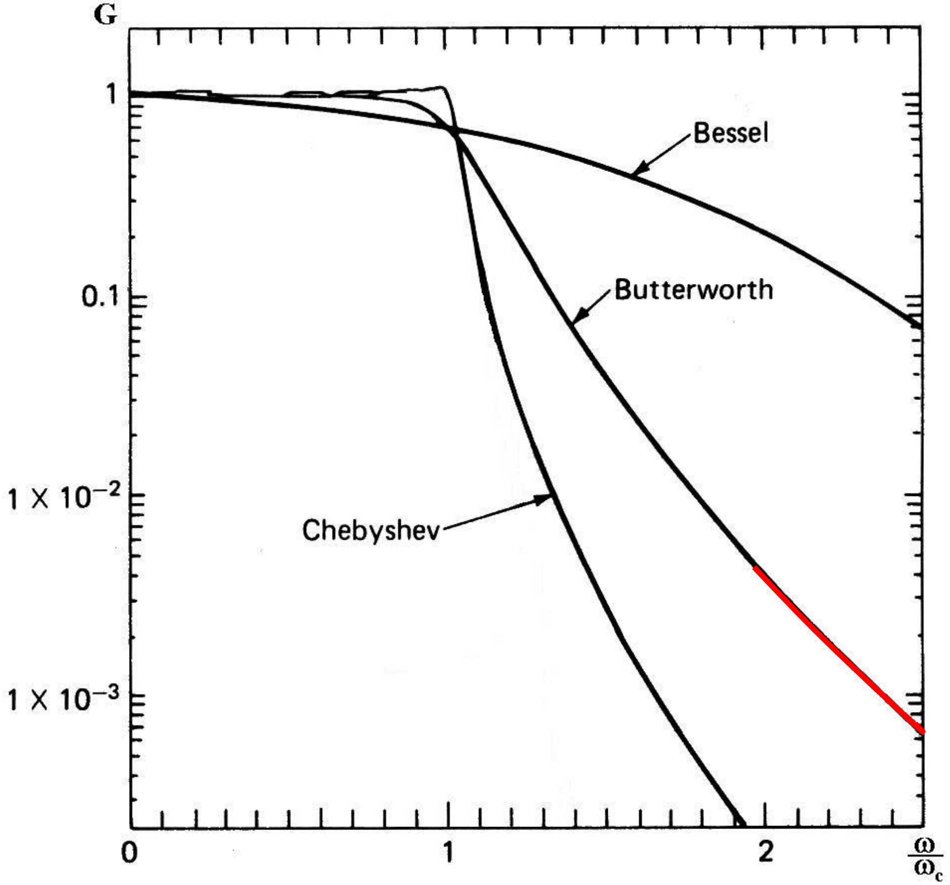
but $X_C = \dfrac{1}{j\omega C}$ is the ***capacitive reactance***, therefore the output/input ratio is : $\dfrac{v_o}{v_i} = \dfrac{1}{j\omega RC + 1}$

Considering $\omega_c = \dfrac{1}{RC} = \dfrac{1}{\lambda}$ the output/input ratio is also: $$\boxed{\dfrac{v_o}{v_i} = \dfrac{1}{j\omega\lambda + 1}}$$

We can calculate the modulus of the previous complex function : $G = \frac{v_o}{v_i} = \frac{1}{\sqrt{\left(\frac{\omega}{\omega_c}\right)^2 + 1}}$ the ***gain*** !

And the phase delay: $\varphi = arctg(-\omega\lambda)$

We recognize the ***frequency response*** of a simple ***1st order electric signal manipulation*** stage !



for ω = 0 $G = 1$

ω → ∞ $G = 0$

ω = ω_c $G = \frac{1}{\sqrt{2}}$

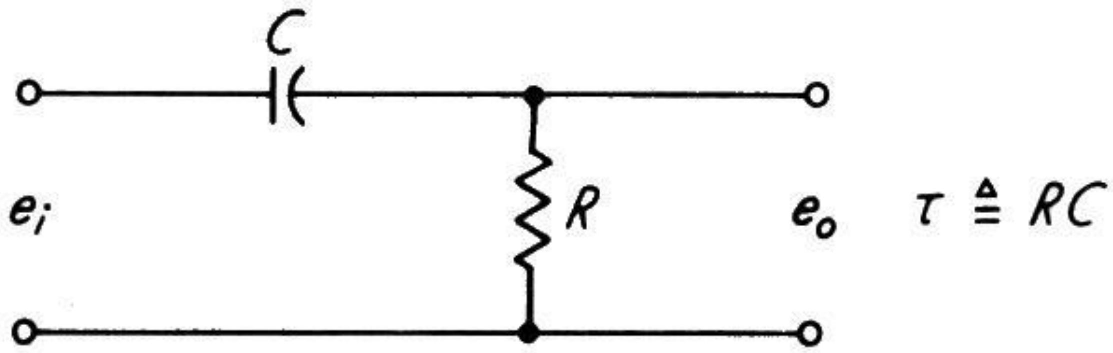
The RC circuit is a ***low-pass filter*** (***one pole Butterworth filter***) with a – 3 db *cut-off frequency*: $f_c = \frac{1}{2\pi RC}$

Note that for $|j\omega CR| >> 1$ which is $\omega >> \frac{1}{CR} = \omega_c$

the *output signal* is also the "integral" of the input signal:

$$v_o(t) = \frac{1}{\lambda} \cdot \frac{1}{j\omega} v_i(t)$$

The high-pass "CR" circuit



The *two electric circuits* equations now are:

input (voltage) signal: $v_i = (X_C + R)\cdot i$

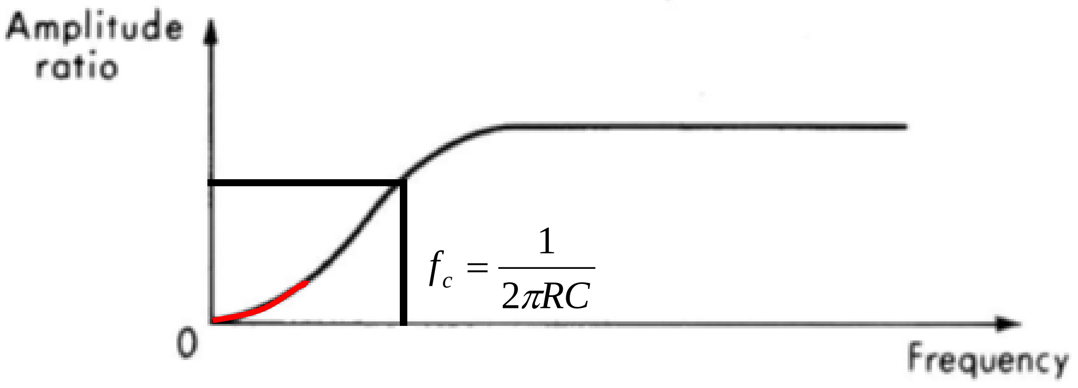
output (voltage) signal: $v_o = R \cdot i$

which now result in : $v_o = R \cdot \dfrac{v_i}{X_C + R}$ with the same notations: $X_C = \dfrac{1}{j\omega C}$ $\omega_c = \dfrac{1}{RC} = \dfrac{1}{\lambda}$

The output/input ratio is : $\dfrac{v_o}{v_i} = \dfrac{R}{X_C + R} = \dfrac{j\omega CR}{1 + j\omega CR} = \dfrac{j\omega\lambda}{1 + j\omega\lambda}$

It is also a ***1st order electric signal manipulation stage*** with a ***gain*** of :

$$G = \frac{v_o}{v_i} = \frac{\dfrac{\omega}{\omega_c}}{\sqrt{\left(\dfrac{\omega}{\omega_c}\right)^2 + 1}}$$



High-pass filter

For $\left| j\omega CR \right| << 1$ or $\omega << \frac{1}{CR} = \omega_c$

we get $G = \frac{v_o}{v_i} \cong j\omega CR << 1$

or $v_o(t) = \lambda \cdot j\omega \cdot v_i(t)$ the *output signal* is the "derivative" of the *input signal* !

$\left(\frac{1}{j\omega}\right)$ → is an ***integration operator***

$(j\omega)$ → is a ***derivative operator***

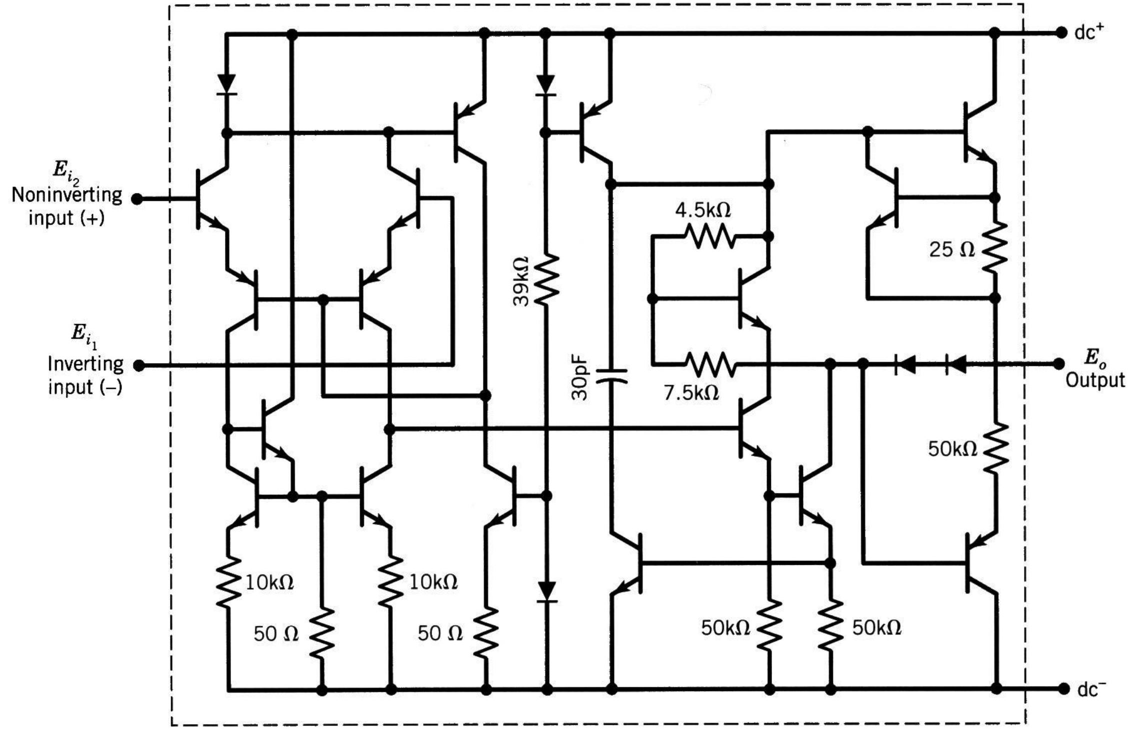
if we consider, for example, a sinusoidal signal

$v_i(t) = V sen\omega t = Ve^{j\omega t}$ and we *derivate* and *integrate* it :

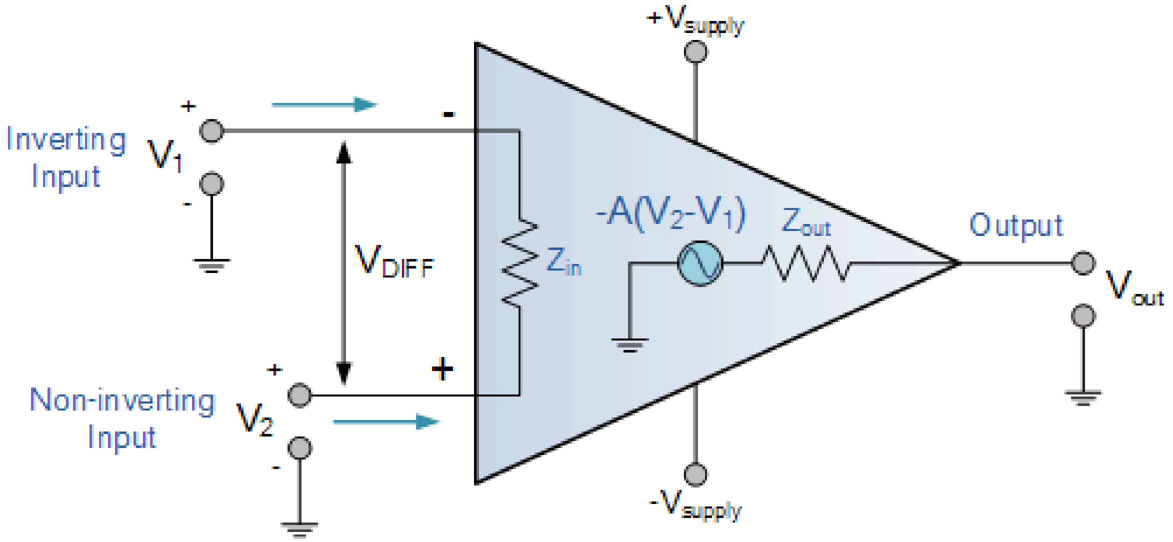
$$\frac{dv_i(t)}{dt} = \frac{dVe^{j\omega t}}{dt} \; j\omega \cdot Ve^{j\omega t} = (j\omega) \cdot v_i(t)$$

$$\int v_i(t)dt = \int Ve^{j\omega t}dt = \frac{V}{j\omega} \cdot e^{j\omega t} = \left(\frac{1}{j\omega}\right) \cdot v_i(t)$$

The operational amplifier OA :



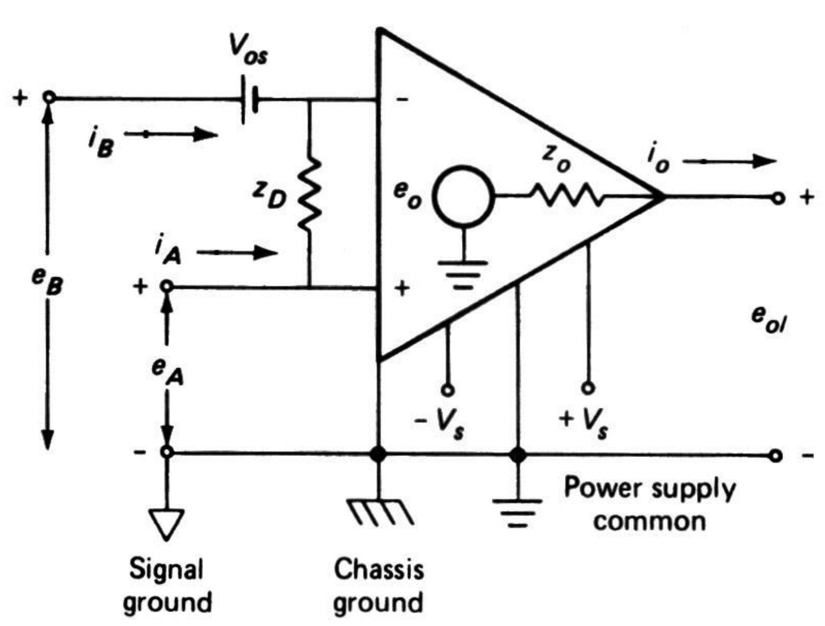
Type 741 circuit diagram

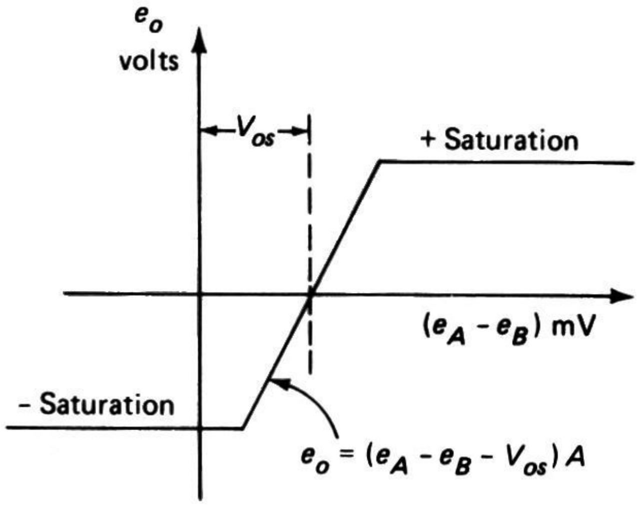


We will <u>not</u> study the inner electronic circuits of the OA but the important *<u>operational features</u>* it offer for the measurements !

Ideal OA characteristics :

Amplification	$A \to \infty$ (10^7)
Input impedance	$Z_i \to \infty$ $(10^{10}\,\Omega)$
Otput impedance	$Z_o \to 0$ $(100\,\Omega)$
Band width	$BW \to \infty$
Offset voltage	$V_{io} \to 0$





The open loop OA can amplify a voltage signal $V_o = A \cdot (V_+ - V_-)$ but NOT over its supply voltages !

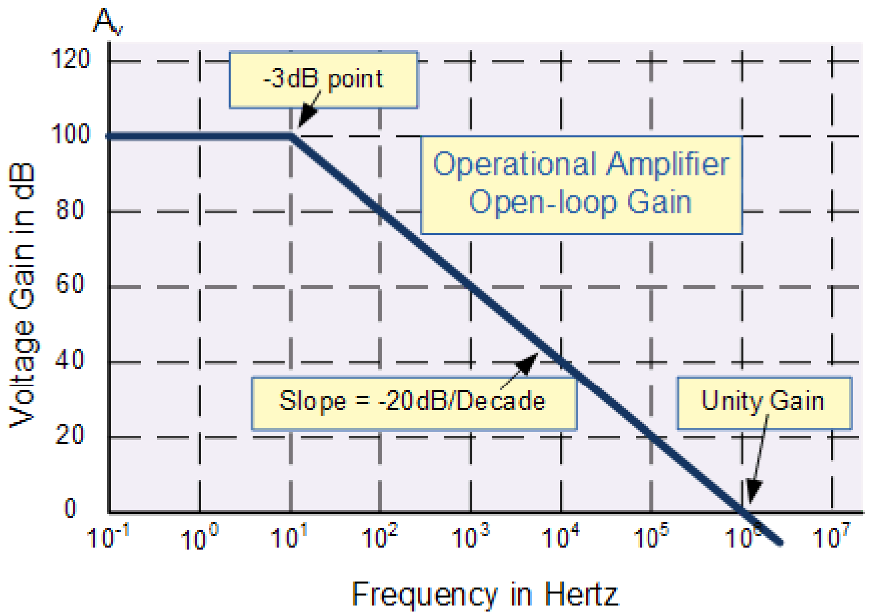
If we have $V_{cc} = \pm 10V$ and $A = 10^7$

Then $\pm 10V = 10^7 \cdot (V_+ - V_-)_{MAX}$

and $(V_+ - V_-)_{MAX} = \frac{20V}{10^7} = 2\mu V$ is the maximum input signal the OA can handle!

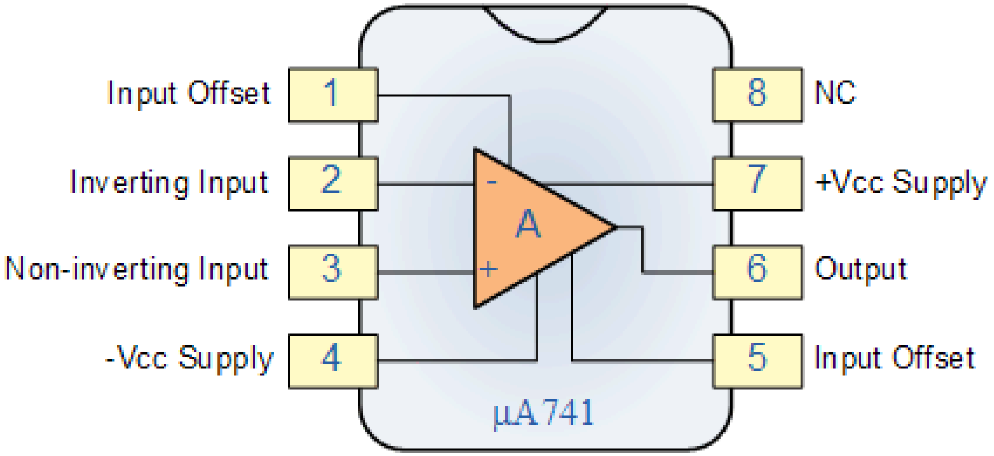
The OA goes into *saturation* for:

$$|V_i| = |V_+ - V_-| > 2\mu V$$



The real *open-loop frequency response* curve is determined by the ***Gain Bandwidth Product (GBP)*** …

$$20\,log(A)\ \ or\ \ 20\,log\frac{V_{out}}{V_{in}}\quad in\ dB$$



The Operational Amplifier CAN NOT be used as an amplifier in the *open loop configuration* !

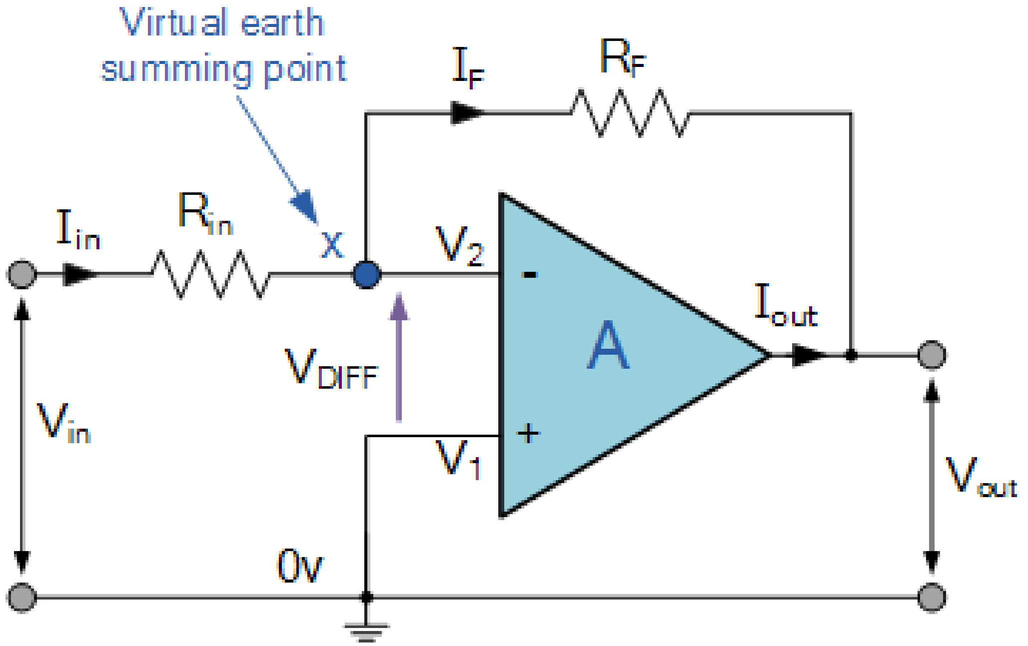
Therefore, we must *connect two resistors* at the terminals as showed in the figure, realizing the ***inverting operational Amplifier*** configuration:

At the virtual earth summing junction we have:

$$V_- \cong V_+ = 0$$

input signal: $V_i = R_i \cdot i_i$

output signal: $V_o = R_f \cdot i_f$



The current at the virtual earth point is: $i_i + i_f = 0$

And because $i_f = -i_i$ we have

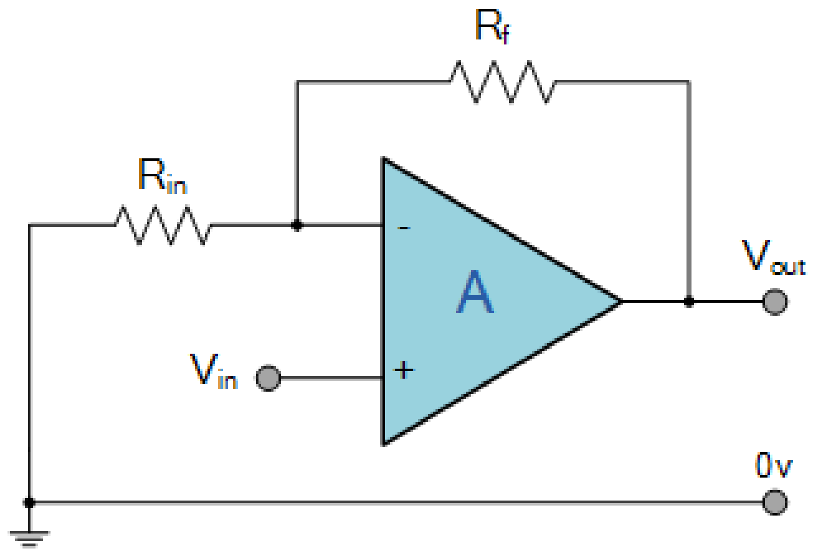
$$V_o = -R_f i_i = -R_f \frac{V_i}{R_i}$$

which is the ***static characteristic curve*** of the device and gives also the ***GAIN*** of the amplifier:

$$G = \frac{V_o}{V_i} = -\frac{R_f}{R_i}$$

which is ***negative !***

To get a "positive" signal amplification we have to switch to the ***non-inverting operational amplifier*** configuration:



Because of the "high input impedance" of the OA device, there is *virtually no current* entering the + terminal of the OA and we have now: $V_- \cong V_+ = V_i$ and $i_f = i$

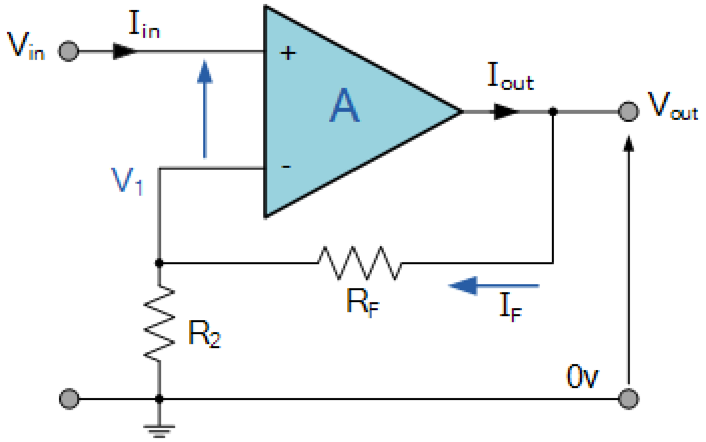
So, the output signal can be written as $V_o = V_f + V_i$
where $V_f = R_f \cdot i_f$ and $V_i \cong V_- = R_i \cdot i$

In the end we have: $V_o = R_f i_f + V_i = R_f i + V_i = R_f \dfrac{V_i}{R_i} + V_i$

And the ***static characteristic curve*** now is:

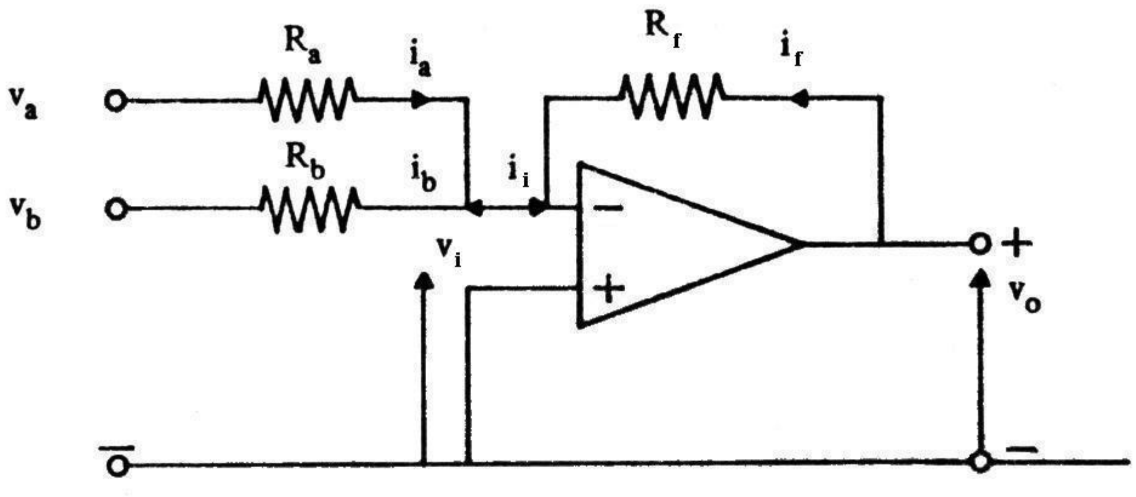
$$V_o = V_i \left(\frac{R_f}{R_i} + 1 \right)$$

with a ***positive GAIN*** !



Both configurations have *much lower amplification* than $10^6 - 10^7$ however the amplification "can be designed" by choosing the two resistance values !
Even if the non-inverting configuration may seem preferable, the inverting configuration has important applications …

The voltage summing OA :



Here we wish to sum two voltages V_a and V_b

From the inverting OA characteristic curve, we have:

$$V_o = -\frac{R_f}{R_i} V_i = -R_f i_i$$

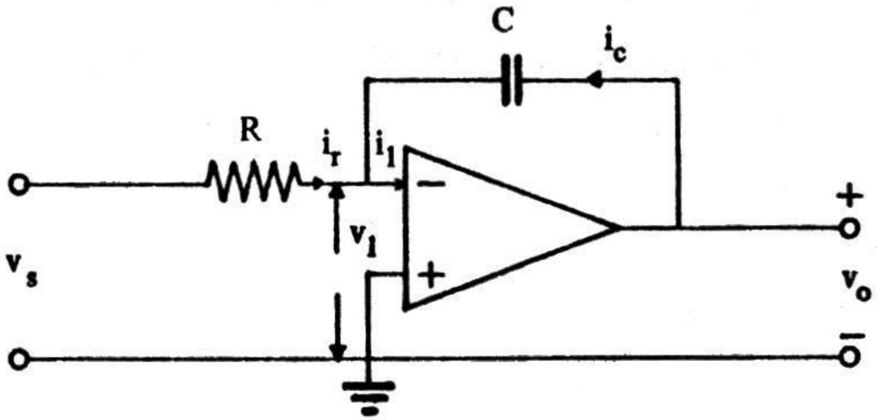
with $$i_i = \frac{V_i}{R_i} = i_a + i_b = \frac{V_a}{R_a} + \frac{V_b}{R_b}$$

If we design the circuit inputs with the two resistance values being equal: $R_a = R_b = R$

We can simplify the above equation and we obtain : $$V_o = -\frac{R_f}{R}(V_a + V_b)$$ which is the ***sum of the two input voltages*** !

Note that summing two voltages means physically to sum the two input currents !

The active low-pass filter (integrator) :



The gain of the inverting OA is: $$v_o = -\frac{Z_f}{Z_i} v_i$$

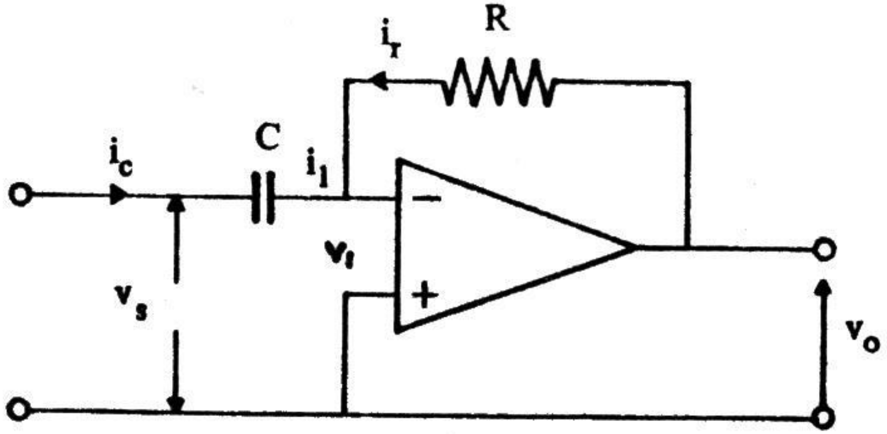
with $Z_i = R_i$ and $Z_f = X_{Cf} = \frac{1}{j\omega C_f}$

Therefore $$v_o = -\frac{1/j\omega C_f}{R_i} \cdot v_i = -\frac{1}{j\omega} \cdot \frac{1}{C_f R_i} v_i = -\frac{1}{j\omega} \cdot \omega_c \cdot v_i$$

the *output signal* is the *electric integral* of the *input signal !*

If we wish to amplify the output, then we have to put an extra resistance R_f on the feedback arm and the extra gain will be : $G = R_f / R_i$

The active high-pass filter (derivative) :

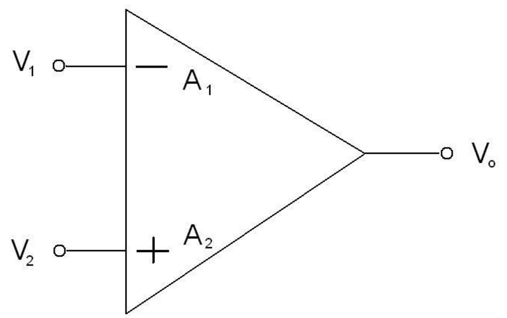


The gain is still: $$v_o = -\frac{Z_f}{Z_i} v_i$$

with $Z_i = X_{Ci} = \frac{1}{j\omega C_i}$ and $Z_f = R_f$

so $$v_o = -\frac{R_f}{1/j\omega C_i} v_i = -j\omega \cdot C_i R_f \cdot v_i = -j\omega \cdot \frac{1}{\omega_c} \cdot v_i$$

the *electric derivative !*



Consider now that the OA accepts TWO voltages at the input terminals $V_+ = V_2$ and $V_- = V_1$ both referred to earth, and amplifies its difference:
$V_o = A(V_2 - V_1)$ … But this is true in the IDEAL case !
In the reality, there are TWO slightly different amplifications at the inputs:
$V_o = A_2 V_2 - A_1 V_1$ …

This situation can be effectively described by the ***differential input*** : $V_d = V_2 - V_1$ and the ***common mode input*** : $V_c = \frac{V_1 + V_2}{2}$ which is the mean distance from the earth reference of the two input voltages.

For an ideal OA we would have $A_1 = A_2 = A$ and the OA would amplify only the differential input, completely eliminating the common mode input !

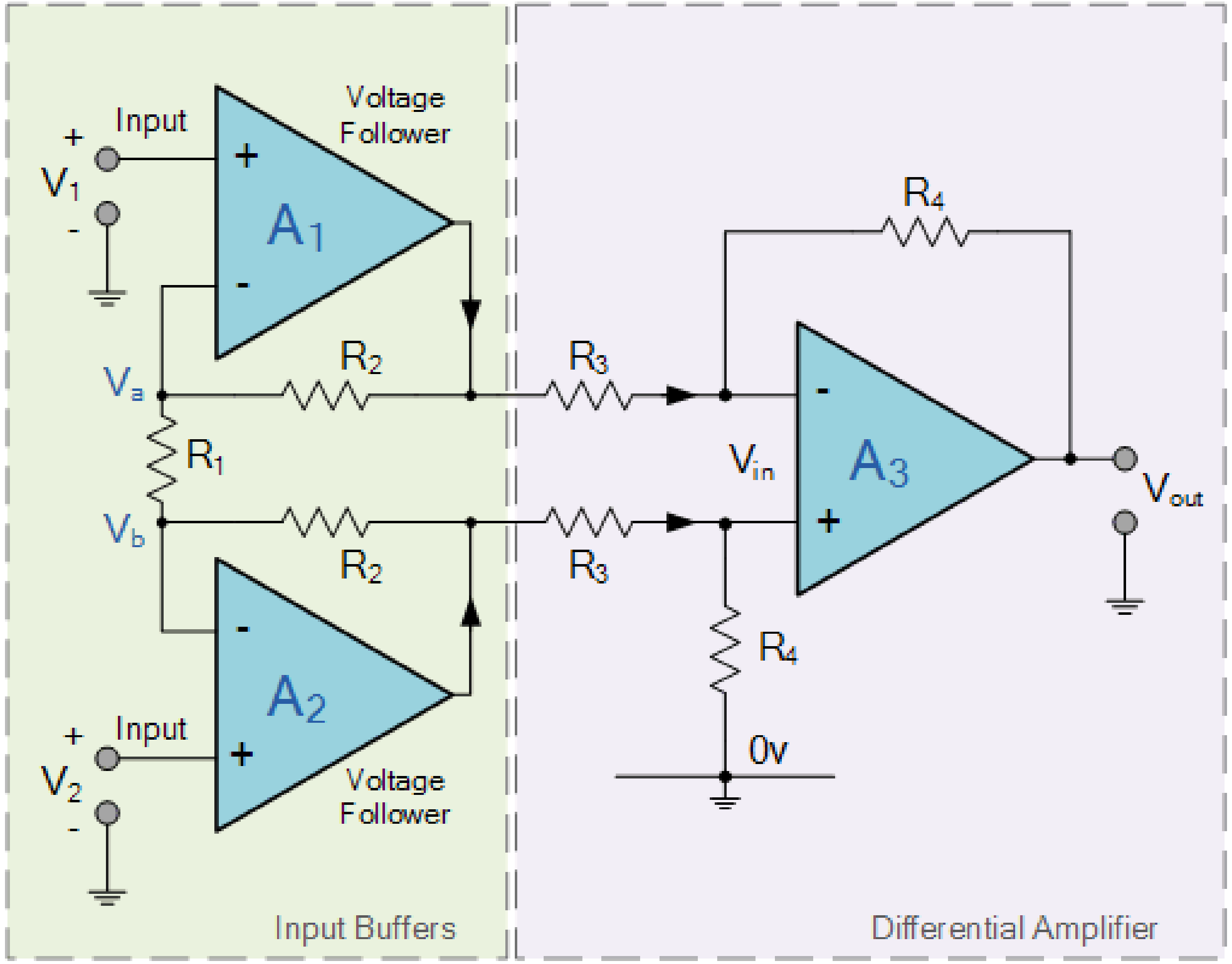
This situation can be effectively described by the ***differential amplification*** : $A_d = \frac{A_1 + A_2}{2}$ which operates on V_d and by the ***common mode amplification*** : $A_c = A_2 - A_1$ which operates on V_c
To minimize this problem the OA must be designed with a high A_d and a very small A_c which means $A_1 \approx A_2$

The ratio between the two amplifications is an important quality parameter of the OA, the ***Common Mode Rejection Ratio*** **:** $CMRR = \frac{A_d}{A_c}$ often expressed in logarithmic scale

$$CMRR = 20 \log \frac{A_d}{A_c}$$

Values range from 60 dB up to 120 Db for high quality OA …

The Instrumentation Amplifier IA :



The most employed amplifier in measurements is designed with two stages:

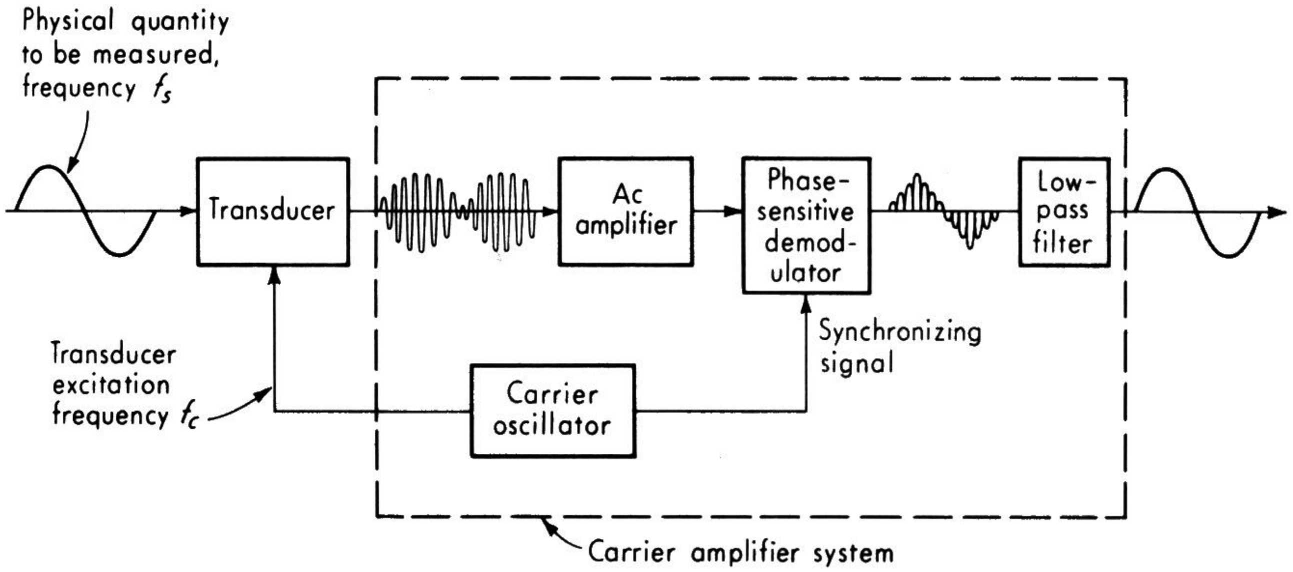
1. A very high input impedance stage ($10^{10}\ \Omega$)
2. A differential amplification stage

It has generally a very high ***Common Mode Rejection Ratio*** (> 100 dB) which makes it suitable to amplify *floating signals* $(v_2 - v_1)$ …

The (differential) ***GAIN*** is :

$$G = \frac{v_o}{v_i} = \frac{R_4}{R_3}\left(1 + \frac{2R_2}{R_1}\right)$$

The Carrier AC Amplifier :



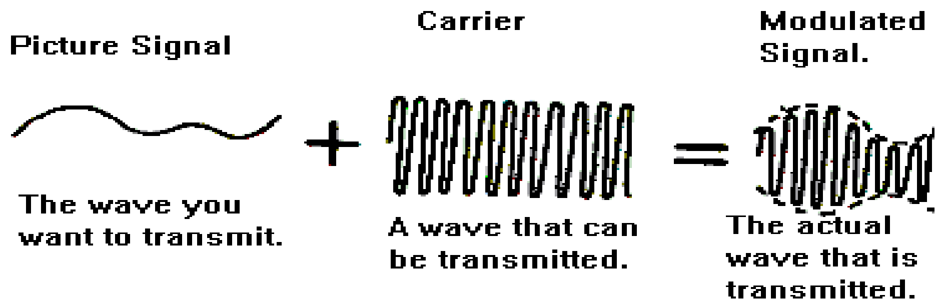
The ***Carrier Amplifier*** is an AC amplifier which does NOT amplify DC signals or components !

It is designed on the "signal modulation and demodulation" principle of radio transmission !

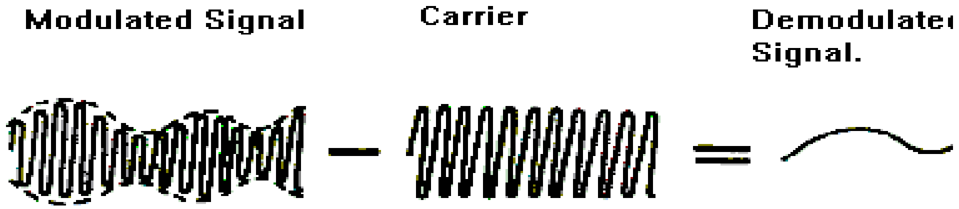
The low frequency (f_s) signal is modulated by the carrier frequency (f_c) which is the only frequency the amplifier is able to amplify !

A *phase sensitive demodulator* and a *low pass-filter* return an amplified low frequency signal !

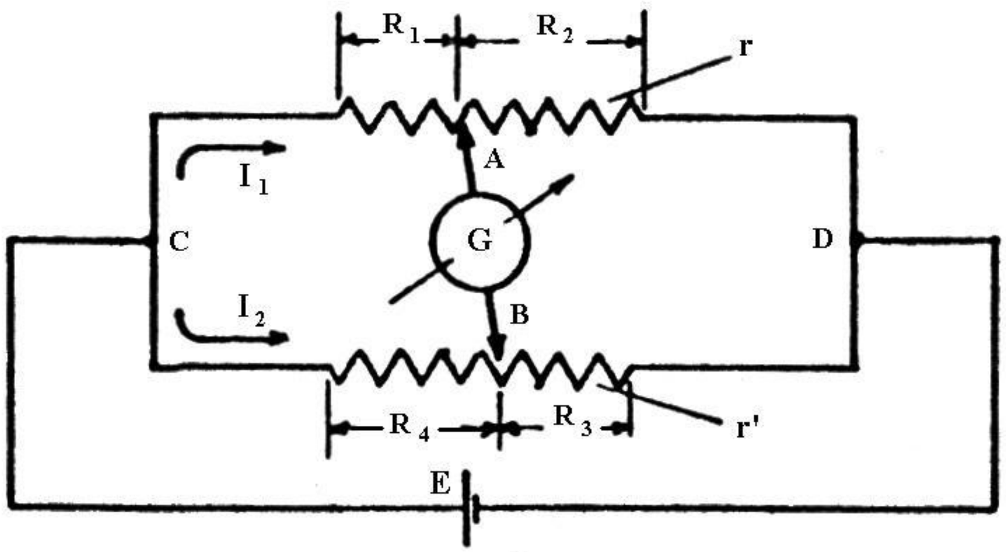
Modulation



Demodulation



The Wheatstone Bridge :



Two resistances *r* and *r'* are connected in parallel and a *galvanometer G* is connected in a bridge configuration across the two resistances in the points A and B …

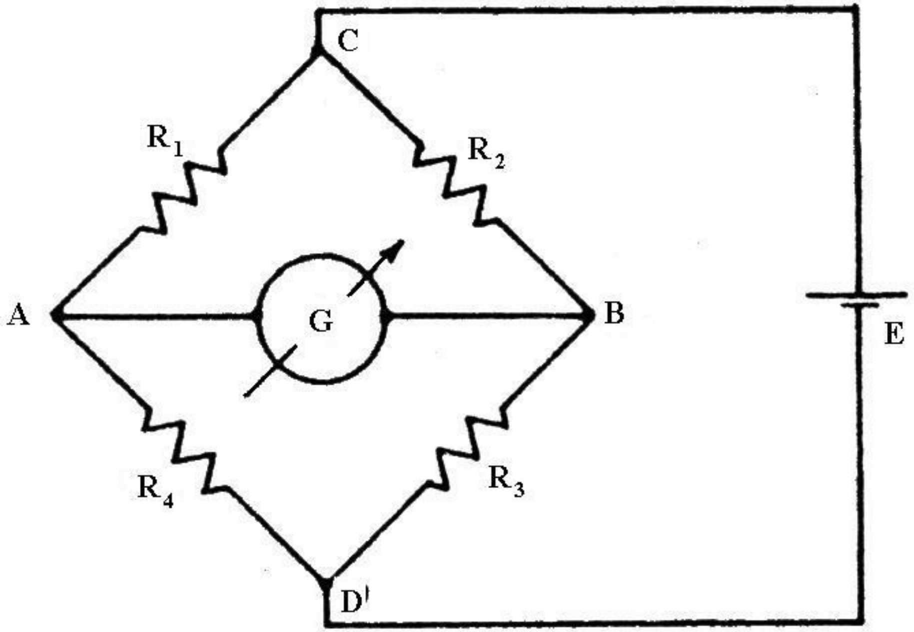
We move now the point A and B on the resistances so to have zero current though the galvanometer, which means also: $V_A = V_B$

In this situation we identified 4 resistances : R_1, R_2, R_3, R_4 for which we can write the following equations:

$$V_{CA} = V_{CB} \quad \rightarrow \quad R_1 I_1 = R_4 I_2$$
$$V_{AD} = V_{BD} \quad \rightarrow \quad R_2 I_1 = R_3 I_2$$

Making the ratio of the two equations, we get: $\frac{R_1}{R_2} = \frac{R_4}{R_3}$ or $\boxed{R_1 R_3 = R_2 R_4}$ the *bridge equilibrium equation* !

No current through the bridge means also $V_A - V_B = 0$ therefore, in practical application, we can substitute the *galvanometer* with a *millivoltmeter* !



The ***Wheatstone Bridge*** is a "zero method" electric network used for resistance measurements:
If $R_x = R_1$ is unknown, we equilibrate the bridge by operating on R2 and, knowing the values of R_2, R_3, R_4, we get:

$$R_x = R_1 = \frac{R_2 R_4}{R_3}$$

However, this is NOT the main use of the Wheatstone Bridge …

We might have a resistance R_1 that changes slightly its value ΔR_1 for physical reasons …

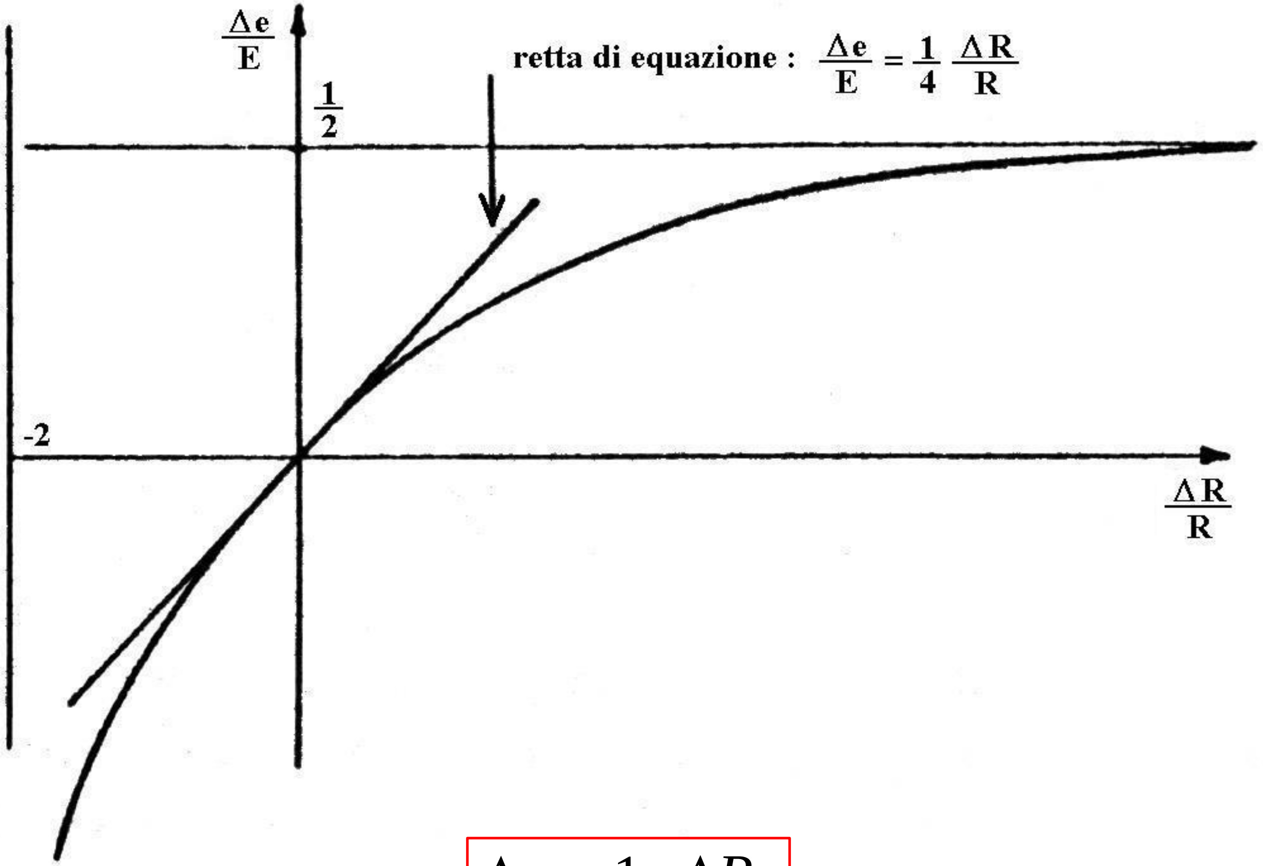
It is possible to read this small change directly on the voltmeter indicator but, to do so, we need to know the ***bridge graduation curve*** (or <u>static characteristic equation</u>): $V_{AB} = f(\Delta R_1)$

If we consider for simplicity a bridge with 4 equal resistances: $R_1 = R_2 = R_3 = R_4 = R$ and a variation ΔR_1 only on the resistance R_1, we get:

The complete ***Wheatstone Bridge characteristic curve*** :

$$\frac{\Delta e}{E} = \frac{\frac{1}{4} \cdot \frac{\Delta R_1}{R}}{1 + \frac{1}{2} \cdot \frac{\Delta R_1}{R}}$$

which is clearly NON linear !



However, if the variation $\Delta R_1 << R$ is really very small: $\frac{\Delta R_1}{R} < 0.01$

Then, the denominator of the graduation curve can be approximated with "1"

$$\frac{1}{2} \frac{\Delta R_1}{R} << 1$$

and the *characteristic equation has been linearized* !

$$\frac{\Delta e}{E} \cong \frac{1}{4} \cdot \frac{\Delta R_1}{R}$$

Linearized ***Wheatstone Bridge*** *characteristic curve* !

Same result if the only resistance with a small variation ΔR_2 would have been R_2 on the second arm of the bridge:

$$\frac{\Delta e}{E} \cong -\frac{1}{4} \cdot \frac{\Delta R_2}{R}$$

negative because R2 is in the negative term of the equilibrium equation: $R_1 R_3 - R_2 R_4 = 0$

In the case of simultaeous small variations of all 4 resistances of the Wheatstone Bridge , we would get:

$$\frac{\Delta e}{E} = \frac{1}{4}\left(\frac{\Delta R_1}{R} - \frac{\Delta R_2}{R} + \frac{\Delta R_3}{R} - \frac{\Delta R_4}{R}\right)$$

The ***Full Wheatstone Bridge characteristic curve*** ! (note the sign alternations … very important property …)